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THE LONG-MEMORY OF NEWSPAPERS' SUBSCRIPTIONS: BETWEEN THE SHORT-RUN AND PERSISTENCE RESPONSE*

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Abstract

The mainstream of marketing time series analysis has shifted from classical short-range dependence (ARMA, transfer functions and VAR models) to persistence models (unit-roots or integrated models, cointegration and error correction models). However, in cases where purchase decisions entail some commitment (e.g., a subscription selling periodic use of a product or service), sales response entails a long-term effect but this effect is not permanent. Long-memory assumes that shocks to a time series have neither a persistent nor a short-run transitory effect, but that they last for a long time and decay slowly with time. Many marketing policies face a short-memory response at the individual customer level but display a considerable degree of persistence at the aggregate level. The aggregation of short-run individual decisions made by heterogeneous customers can show a long-memory pattern. In today's highly competitive newspaper industry, loyal, ongoing customers are a key to obtain stable and long-term profits. Often newspapers obtain a loyal customer base through subscriptions. This paper proposes a long-memory model to study the long-term sales response dynamics in subscription markets. The model accounts for the heterogeneity of the individual responses and distinguishes between both trend and long-memory components pattern of subscriptions. This model permits more accurate predictions of subscription sales than those obtained using persistence models.

Keywords: Long-memory, Persistence, Time Series, Subscription markets, Newspapers
JEL Classification: M3, C22, C53

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INTRODUCTION

In many sectors, company sales evolve over time accumulating the effect of previous shocks in a persistence pattern, and this evolution translates in return trends. However, not all markets show clear trends. Some reach a stationary equilibrium where the effect of previous shocks fades quickly in a short memory pattern (Dekimpe and Hanssens, 1995). The dichotomy between short memory and persistence is nowadays a central issue in market sales response analysis. However, both short memory and persistence modeling are of limited use in the context of those markets in which there are a reasonable number of loyal customers, such as subscriptions. Short-memory modeling assumes that the effect of fluctuations in sales fades in just a few periods, whereas persistence models impose that the impact of fluctuations in sales have an everlasting impact (e.g., in the case of subscriptions, this assumption implies that once a customer subscribes to a newspaper, its subscription lasts for ever). However, in this paper we argue that these two hypotheses are unrealistic, since the impact of an improvement in subscription sales has a long-range effect, but not so large that it can be assumed as permanent.

We build a long-memory model to forecast future subscriptions, and the results show that the forecasting accuracy of the long-memory model is clearly superior to its persistent counterpart, the unit root model. Our research draws on prominent issues in sales response analysis. In particular, this paper enables us to answer the following questions:

- Does sales response exhibit a short-memory, long-memory or persistence pattern in a particular market? We consider long-memory modeling to describe long-range sales responses. The potential of using long memory is acknowledged as a tool that nests all time series approaches; namely, short, long and persistence memory. The degree of “memory” of a time-series model is determined by a parameter and we describe an estimation procedure to contrast our hypotheses. This parameter provides a measure of retention in the time-dependence of subscription sales.
- May the aggregation of a large number of heterogeneous individual responses exhibit long-range dependence, even though there is no dependence at an individual level? Following Robinson (1978), we study this behaviour in the

context of subscription markets. In particular, we show how the proposed model captures the main features of individual commitment to different newspapers.

- Finally, do long-memory models provide a powerful tool in predicting future market responses? Our findings suggest that the considered long-memory model performs well in forecasting the future subscriptions of three Spanish newspapers, and shows a clear superiority to its persistent counterpart, the unit root model.

The paper is structured as follows. First, we introduce the concept of long-memory processes. We then present a long-memory model of customer response behavior in subscription markets. The next section presents the empirical application and summarizes our main findings. Finally we present some concluding remarks.

LONG-MEMORY PROCESSES: BETWEEN PERSISTENCE AND SHORT-RUN DEPENDENCE

The contribution of Time-Series (TS) methods to Marketing Science has become of increased interest in recent years (see Dekimpe and Hanssens, 2000, 2004a, and Hanssens, Parsons and Schultz, 2001). Marketing literature has used TS methods for various purposes, which include forecasting (Geurts and Ibrahim, 1975), crossed dynamic effects between marketing mix and performance response variables (Helmer and Johansson, 1977), causality tests (Leeflang and Wittink, 1992), and marketing policy simulations (Franses, 2005) . Initially, the marketing literature was concerned with quantifying short-term effects, and most of the literature considered short-memory TS models, such as univariate ARMA models, ARMAX or transfer functions, and multivariate VAR or VARX models (see, e.g., Dekimpe and Hanssens 1995b, Pauwels, Hanssens and Siddarth 2002). In all of these models the autocorrelation (a measure of dynamic dependence) fades very quickly (in other words, all of these models have short-memory). In recent times the focus has shifted from short-range effects into long-term and particularly persistent effects (see e.g., Mela, Gupta and Lehmann, 1997, Dekimpe and Hanssens 1995b, 1999, 2004b). Persistence is usually tested by unit-root tests, and in this context the variables are modeled as an ARIMA model, and for multivariate relationships the most common specifications are integrated or cointegrated VAR models and error-correction models (see ,e.g., Cavaliere and Tassinari, 2001).

Often sales time series entail a long-term effect, but usually this effect is not permanent. Long memory assumes that shocks to time series have neither a persistent nor a short-run transitory effect, but that the statistical dependence decays slowly with time. Whereas econometric theory was focussed on persistence models from the 80s to the late 90s, nowadays, most of the time series methodological research is focused on long memory models (see e.g. Robinson, 2003), with little echo yet in more applied disciplines like business organization and marketing in particular.

Persistence, long and short-memory time series have been described by the autocovariance function. Assume that data $\{X_t\}$ follows a second order stationary process; i.e. the process has time-stable second order moments (with common mean $\mu = E[X_t]$ and autocovariances function $\{\gamma_l\}_{l=-\infty}^{+\infty}$, where $\gamma_l = Cov[X_t, X_{t+l}]$ for all *t*). The property $\sum_{l=-\infty}^{+\infty} |\gamma_l|^2 < \infty$ is satisfied by all stationary processes, implying that $\lim_{l \to \infty} \gamma_l = 0$ (i.e. the autocovariances tend to zero, but the rate of convergence can be slow). Then we distinguish two situations: (1) We say that the stationary time series $\{X_t\}$ has short memory if, in addition to stationarity, the autocovariances satisfy $\sum_{l=-\infty}^{+\infty} \gamma_l < \infty$ (the autocovariances tend to zero at a fast rate); (2) We say that $\{X_t\}$ has long memory if, regardless of the stationarity, the rate of convergence is so slow that $\sum_{l=-\infty}^{+\infty} \gamma_l = \infty$. Essentially, there is long memory if for some $c_2 > 0$ and $d \in (0,0.5)$, $\gamma_l \approx c_2\, l^{2d-1}$ as $l \to \infty$. If $d = 0$, then there is short memory, and if $d > 0.5$ the process is not stationary ($\sum_{l=-\infty}^{+\infty} |\gamma_l|^2 = \infty$).

ARMA models are not appropriate for modeling long-memory processes because the covariate structures associated to these processes tend to zero very quickly. Persistence (unit root) models are also inappropriate for modeling long-memory. Perhaps the simplest model for long memory is the Adenstedt (1965) fractional differencing model of order *d*, given by $(1-L)^d X_t = \varepsilon_t$. Using Newton's Binomial formula, we can formally express,

$$(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)} L^j ,$$

where $\Gamma(\cdot)$ is the Gamma function $\Gamma(\alpha) = \int_0^{\infty} e^{-x} x^{\alpha-1} dx$, (recall that $\Gamma(\alpha) = (\alpha-1)!$ for all non-negative integers α). Similarly, $(1-L)^{-d}$ can be expanded as an infinite order polynomial in L, with coefficients $\Gamma(j+d)/\Gamma(d)\Gamma(j+1)$. The fractional differencing model nests the unit root specification (*d=1*), stationary long-memory (*0<d<0.5*), and short-range memory (*d=0*). Since the long memory models nest the integrated processes, we can test the null hypothesis *d=1* to check if the unit root assumption is satisfied, as an alternative procedure to classical unit root tests. The Adenstedt (1965) model can be generalized to one of the most popular specifications; mainly, the fractional autoregressive integrated moving average model ARFIMA(p,d,q), considered by Granger and Joyeux (1980) and Hosking (1981), given by

$$\Psi_q(L)(1-L)^d X_t = \Phi_p(L)\varepsilon_t ,$$

where $\Psi_q(L) = 1 - \sum_{j=1}^{q} \psi_j L^j$ and $\Phi_p(L) = 1 + \sum_{j=1}^{p} \varphi_j L$ have roots outside the unit circle and have not common roots. For a review see, e.g., the surveys of Beran (1992), Baille (1996), Robinson (1994), and the monographic review edited by Robinson (2003).

MODELLING A SUBSCRIPTION MARKET

In this section we put forward a model for the long-memory customer behavior in subscription markets, such as the newspaper market, using Robinson's (1978) and Granger's (1980) work on long memory.

We assume that subscriptions fall into committed and uncommitted customers. The first ones are strongly motivated by their preferences about newspaper style and political allegiances, in contrast to uncommitted customers who are switching buyers in the medium term. The model is built around the idea that aggregated sales have a deterministic trend and a random component. The trend component typically includes institutional subscriptions and individual customers with such a very strong commitment, and therefore these sales can be regarded as deterministic. The random component includes those sales generated by heterogeneous customers with a moderate commitment. Therefore, for all *t>0*, aggregated sales $\{Y_t\}$ satisfy that

$$Y_t = a + bt + w_t, \qquad (1)$$

where the random term w_t, is determined by the aggregation of subscriptions of individuals with moderate commitment. For each one of these individuals i, commitment is an unobservable latent y_t^i defined as a continuous one-dimensional variable in the real line which represents a continuum of preferences, with zero point representing non-intention to subscribe to the newspaper. Therefore, increasing y_t^i means increasing purchase intention to subscribe to the newspaper. If we aggregate the individual subscription intentions in this market, we obtain the latent sales given by $w_t = \sum_{i=1}^{N} y_t^i$, at the period of time *t*.

Assuming that the individual impulse y_t^i to subscribe to a newspaper is a latent variable which depends only on the previous intention, we model the decision process as a stationary first-order auto-regressive model

$$y_t^i = \alpha^i \ y_{t-1}^i + \varepsilon_t^i, \qquad (2)$$

where $\alpha^i \geq 0$ is the autoregressive parameter, and $\{\varepsilon_t^i\}$ is a series of independent random shocks with zero-mean and constant variance σ_ε^2 / N, (that we regard as normalized by N, so that $N^{-1} \sum_{i=1}^{N} Var(\varepsilon_t^i) = \sigma_\varepsilon^2$). Notice that at the individual level we assume that the previous shocks have a temporary effect on the individual purchase intention, i.e. $\alpha^i < 1$ and the impact of past shocks diminishes and eventually becomes negligible. A high value of α^i would mean that the customer is moderately or quite committed to the brand and likely to remain a subscriber, whilst values of α^i close to zero are typical in highly uncommitted customers who do not keep their subscriptions for too long. Then the distribution of α^i represents the distribution of individual attributes for subscriptions in the newspaper market.

Next, we consider that the potential market contains *N* individuals, and $\{\alpha^i : i = 1,..,, N\}$ are independent and identically distributed with a modified *Beta(p,q)* density distribution ,

$$g(\alpha) = 2\frac{1}{B(p,q)}\alpha^{2p-1}\left(1-\alpha^2\right)^{q-1}, \quad 0 \le \alpha \le 1, \tag{3}$$

with $1 < q < 2$, $p > 0$ which is related to the standard Beta distribution by a change of variable $x = \alpha^2$. Then, if the aggregation of heterogeneous individual purchase intentions exhibits long-memory (i.e. dependence between observations at long range). If N is large, the noise process $\{w_t\}$ has autocovariance satisfying that,

$$\gamma_k = \frac{\sigma_\varepsilon^2}{2\pi}\int_{-\pi}^{\pi} e^{ik\lambda} E\left[\left|1-\alpha e^{-i\lambda}\right|^{-2}\right] d\lambda = \sigma_\varepsilon^2 \frac{\Gamma(q-1)}{B(p,q)} \frac{\Gamma(p+k/2)}{\Gamma(p+k/2+q-1)} = O\left(k^{1-q}\right). \tag{4}$$

Comparing with the long-memory process order $\gamma_k = O\left(k^{2d-1}\right)$, it follows that the aggregate impulse to subscribe to a newspaper $\{w_t\}$ has long memory with

$$d = 1 - q/2 \in (0,0.5). \tag{5}$$

The coefficient p does not have an impact on the memory of the process but it helps us to evaluate the proportion of medium-to-high versus low committed subscription customers.

APPLICATION

In this section we empirically evaluate the long-memory subscription behavior of the Spanish newspaper market. We first discuss the data. Then, we estimate the model parameters and study the presence of long-memory behavior in the subscription data. Finally, we use the estimations' results to analyze the subscription market and to forecast future subscriptions.

The data

The data that we use is from a circulation audit office that monitors and certifies the sale of newspapers, large-circulation magazines and technical and professional publications. It is panel data on monthly subscriptions of three daily newspapers in Spain, which are three of the largest national daily newspapers in circulation. Newspapers A and C are focused on the right-wing readers, whilst the market leader is Newspaper B which is a left-wing newspaper. For Newspaper A, we consider a data sequence that begins January, 1994 and ends December, 2004. For Newspaper B, data begins January, 1994 and ends December, 2004. Newspaper C was first published in 1999 and the sample period runs from February, 1999 to December, 2004.

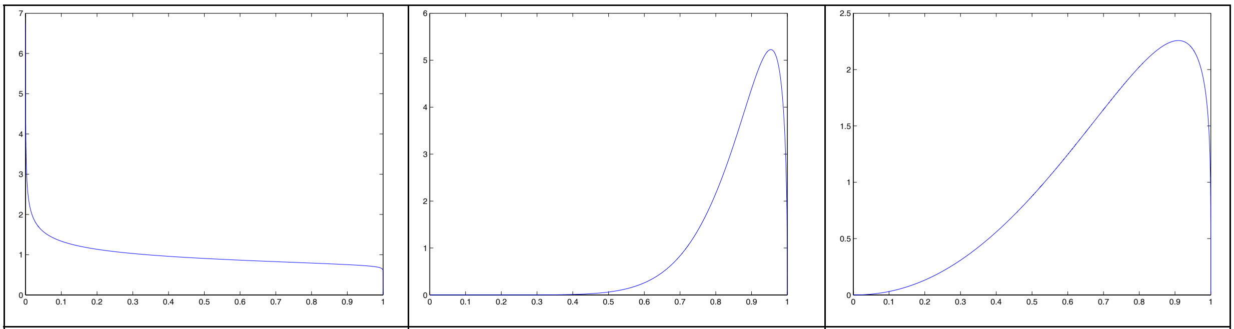
Estimation

To account for general trend pattern of subscriptions, we begin by estimating (1), and using the residuals to estimate the memory of the process $\{w_t\}$. Table 1 reports the estimates of *a* and *b* based on an OLS regression, and the memory parameter estimates *d* for $\{w_t\}$ based on Künsch (1987) and Robinson's (1995) local whittle method. From the results in Table 1, we conclude that the estimates of *d* are in the range $0<d<0.5$, supporting the fact that the customer behaviour in the subscription market of the three newspaper follows a stationary long-memory process.

TABLE 1: Estimation results for three Spanish newspapers

Parameter	**Newspaper A**	**Newspaper B**	**Newspaper B**
a	6990.2	5993.7	168.2423
b	207.85	60.8244	9.9927
d	0.4824	0.2972	0.3885
$\hat{\sigma}^2(a_t)$	1.56e+007	1.722e+005	2114.4
T	109	109	65

Furthermore, we consider the density distribution (3) to evaluate the commitment of the subscription customers for each newspaper. In order to do so, we compute the estimates of the parameters q and *p*. From $\hat{d}$ we can get an estimation of the memory parameter q using (5). To calibrate the parameter *p* in (3), we will consider minimizing the distance between the autocorrelations from the data and the autocorrelations in (4). The solution to this problem is the parameter *p* that best fits the data. Figure 1 shows the shapes of the densities (3) for the three considered newspapers. These shapes represent the distribution of customer commitments for each newspaper. Therefore, we conclude that the proportion of "relatively committed" buyers is the highest for Newspaper B, which is also the market leader, and it is also high in Newspaper C. In contrast, Newspaper A achieves the highest share of "uncommitted" buyers.



Newspaper A	Newspaper B	Newspaper C

Figure 1. Modified beta density function for the three Newspapers.

Forecasting

From the estimates presented in Table 1, we can forecast post-sample values of $\{w_t\}$ and the subscription patterns taking a simple fractional difference model $(1-L)^d w_t = a_t$; i.e. for several integer values $l>0$,

$$E[w_{T+l} \mid I_T] = \sum_{j=l}^{\infty} a_j w_{T+l-j}$$

with $a_j = \Gamma(j-d)/\Gamma(-d)\Gamma(j+1)$ and setting w_j as zero for $j<1$. The forecast error has variance $V_l^2 = \sigma_a^2 \sum_{j=0}^{l-1} b_j^2$, where $b_j = \Gamma(j+d)/\Gamma(d)\Gamma(j+1)$. Note that an AR(p) model with a very large p can resemble the behavior of long-memory models, but the model is not parsimonious and therefore, it reduces the overall efficiency. Long memory processes forecasts can be essentially based on the parameter d .

Figure 2, 3 and 4 present the multistep forecasts of $\{Y_t\}$ generated by the long memory model and a unit root model for the three cases, respectively. It also shows the actual post-sample data which closely resemble the fractional integration forecasts. Two figures are presented for each newspaper. The upper one includes sample and post-sample forecasts; the lower one amplifies the post-sample details. The unit root forecasts follow a linear trend, determined by the persistence of shocks, which is too rigid for these data.

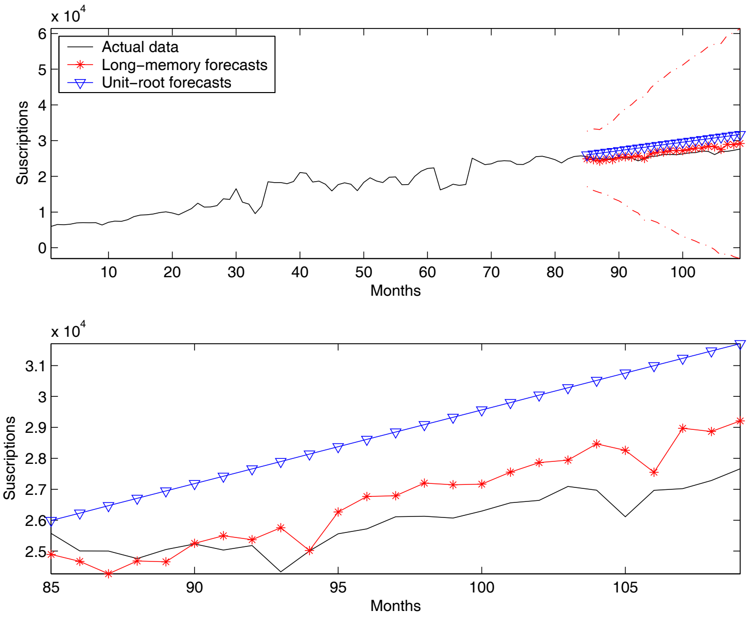


Figure 2. Newspaper A. Post-sample forecasts based on a fractional difference model (with 95% confidence intervals) a unit root model, and actual data.

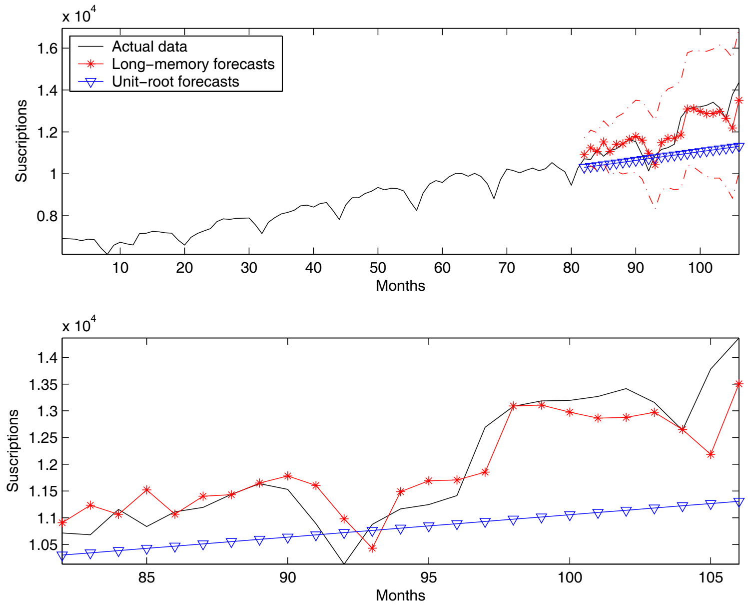


Figure 3. Newspaper B. Post-sample forecasts based on a fractional difference model (with 95% confidence intervals) a unit root model, and actual data.

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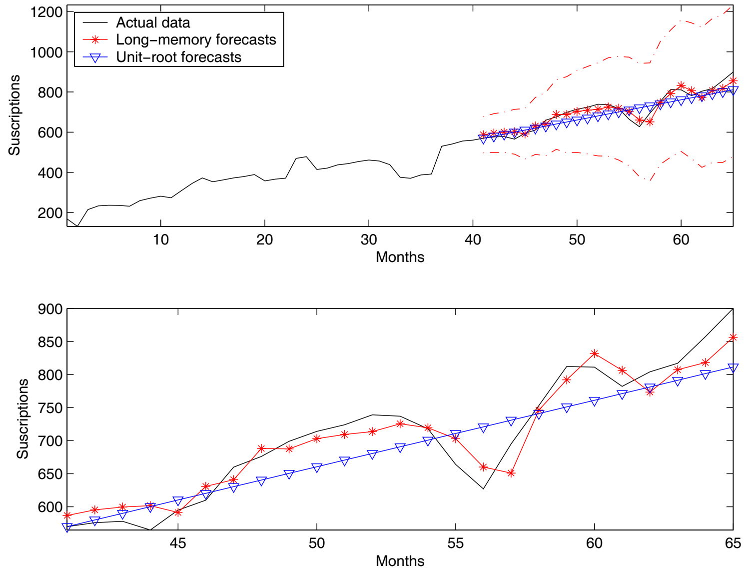

Figure 4. Newspaper C. Post-sample forecasts based on a fractional difference model (with 95% confidence intervals) a unit root model, and actual data.

The above figures show the clear forecast improvements in subscriptions using long memory models compared with classical unit root models. Reducing forecasting errors may lead to cost reductions in the printing and distribution planning.

CONCLUSIONS

The mainstream of marketing literature has recognized the interest in measuring long-term effects of marketing activities (Rust et al. 2004). Dekimpe and Hanssens (1995b) face this challenge modeling the persistence of marketing effects on sales. However, the disjunctive between short-range memory and persistence is too radical: either the impact of previously observed marketing variables on the present values fades very quickly (short-memory) or it remains for ever (persistence). Imposing a permanent response to marketing responses may be sometimes too tight an assumption, and as result, it can bias upwards the expected performance of marketing policies and affect the optimality of resource allocation decisions.

A major challenge for TS modelers in marketing is to expand the scope of TS techniques to allow for intermediate situations. An intermediate view is the “Long-

Memory" approach. Since the question of short-run versus long-run market response to managers' decisions lies at the core of marketing strategy, it is crucial to have a rich variety of models to increase our understanding of these effects, providing guidelines for optimal resource allocation (Dekimpe and Hanssens, 2004b).

This paper considers a framework that nests short-range, long-memory and persistence models in an attempt to explain the long-term dependence of subscriptions in newspaper markets. We present a subscription model that accounts for the heterogeneity of the target population and both trend and long-memory components pattern of subscriptions. Steady trends typically account for demography and growth of segments strongly committed with the newspapers. On the other hand, there are also customers that react to shocks and can vary their subscriptions with a long-term pattern. The distributions of these two types of consumers are depicted in the form of Beta densities. Furthermore, the results show the clear forecast improvements in subscriptions using long-memory models compared with classical unit-root models. The use of long-memory forecasts in media planning provides a competitive advantage derived from a more efficient design of resource requirements and newsprint.

The model and methodology employed in this paper are broadly applicable to other to other industry sectors. Since more businesses are recognizing the importance of brand loyalty to ensure long-term profitability, marketing managers have focused increasingly on creating and maintaining brand loyalty. Subscription is often the best way to ensure reader loyalty. A distinctive characteristic of these types of markets is that transactions are not anonymous, and managers can easily define effective loyalty strategies such as price adjustments based on changing situations. There is a large variety of goods and services sold in a subscription market such as internet access, satellite television, health club services, magazines and newspapers.

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